LAST NAME: ____________

FIRST NAME: Solution

THEORY OF COMPUTATION
CSCI 320, course # 63570
Test # 1

April 1, 2015

instructor: Bojana Obrenić

NOTE: It is the policy of the Computer Science Department to issue a failing grade in the course to any student who either gives or receives help on any test.

Your ability and readiness to follow the test protocol described below is a component of the technical proficiency evaluated by this test. If you violate the test protocol you will thereby indicate that you are not qualified to pass the test.

this is a **closed-book** test, to which it is **forbidden** to bring anything that functions as: paper, calculator, hand-held organizer, computer, telephone, camera, voice or video transmitter, recorder or player, or any device other than pencils (pens), erasers and clocks;

answers should be written only in the space marked "**Answer:** " that follows the statement of the problem (unless stated otherwise);

scratch should never be written in the answer space, but may be written in the enclosed scratch pad, the content of which *will not be graded;*

any problem to which you give **two or more (different) answers** receives the **grade of zero** automatically;

student name has to be written **clearly** on **each page** of the problem set and on the first page of **scratch pad** the during the **first five minutes of the test**—there is a penalty of **at least 1 point** for each missing name;

when requested, **hand in** the problem set together with the scratch pad;

once you leave the classroom, you cannot come back to the test;

your **handwriting** must be legible, so as to leave no ambiguity whatsoever as to what exactly you have written.

You may work on as many (or as few) problems as you wish.
time: 75 minutes.
full credit: 100 points.

Good luck.

problem:	01	02	03	04	05	06	07	08	09	10	11	total: [%]
grade:												

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Problem 1 [**10 points**] Let L be the language defined by the regular expression:

$$(\boldsymbol{baaa})^* \, (\boldsymbol{ad} \cup \boldsymbol{c}^*\boldsymbol{ba})^*$$

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$, $\Sigma = \{a, b, c, d\}$, $V = \{S, A, B, D\}$

P: $S \to AB$

$A \to \lambda \mid AA \mid baaa$

$B \to \lambda \mid BB \mid ad \mid Dba$

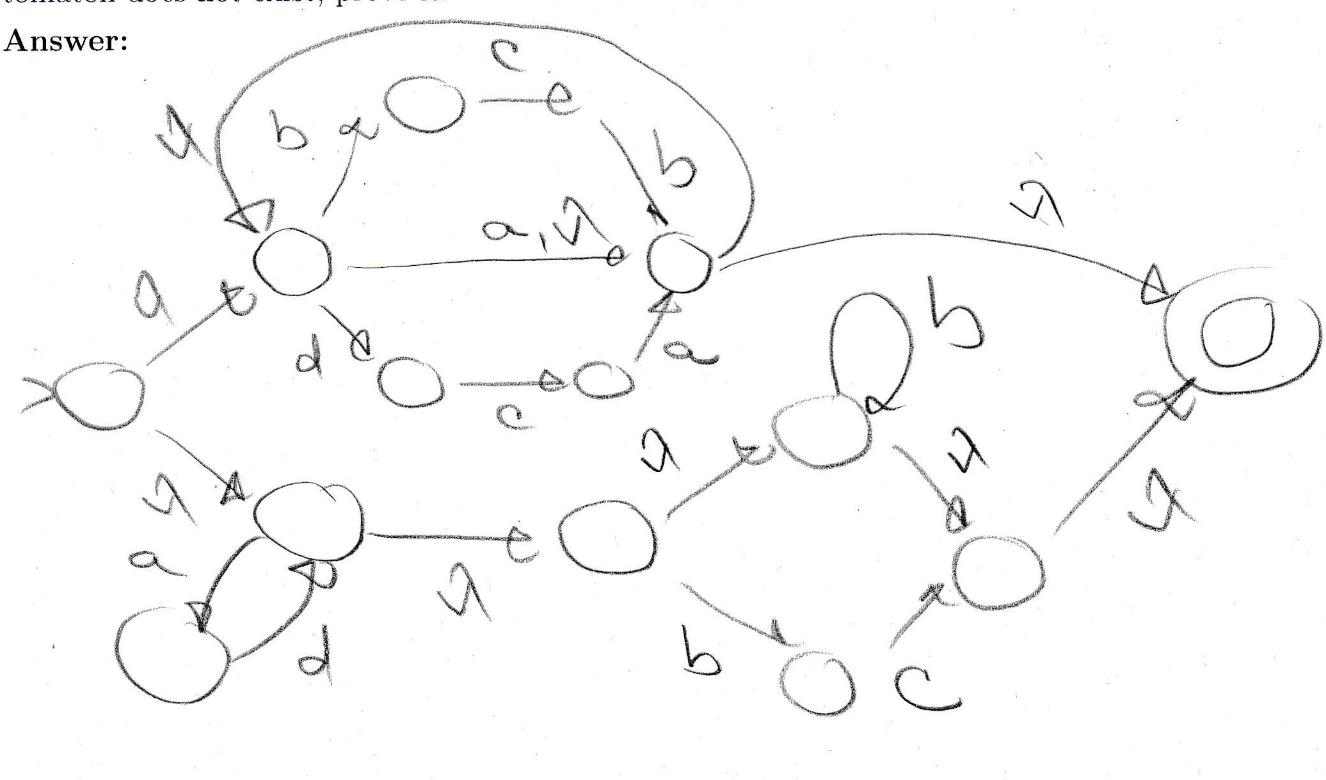
$D \to cD \mid \lambda$

Problem 2 [**10 points**] Let L be the language defined by the regular expression:

$$(\boldsymbol{a} \cup \boldsymbol{bcb} \cup \boldsymbol{dca})^* \cup (\boldsymbol{ad})^* \, (\boldsymbol{b}^* \cup \boldsymbol{bc})$$

Draw a state-transition graph of a finite-state automaton that accepts the language L. If such an automaton does not exist, prove it.

Answer:



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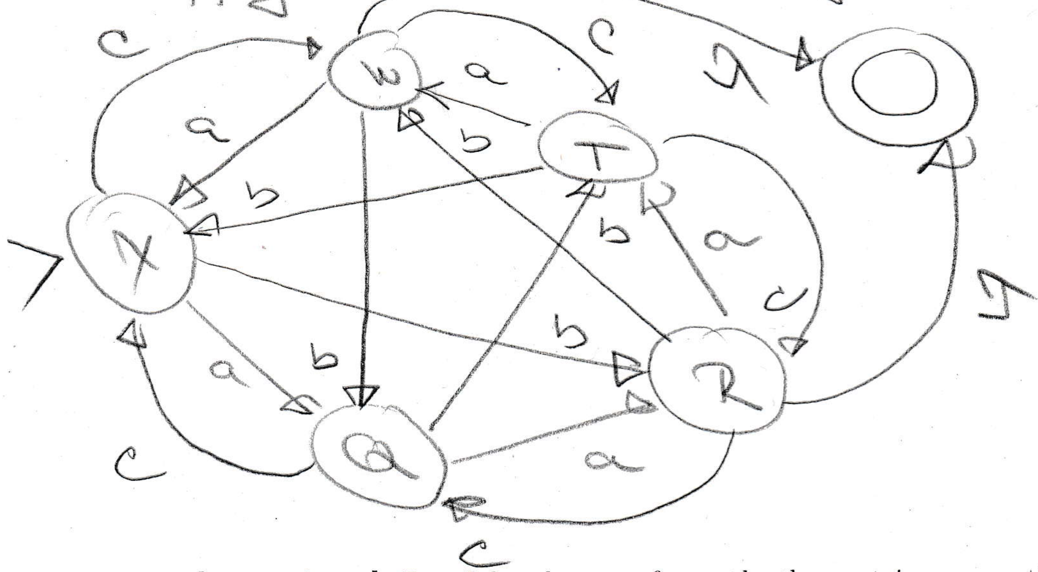
Problem 3 [**10 points**] Let L be the language generated by the context-free grammar $G = (V, \Sigma, P, X)$, where $\Sigma = \{a, b, c\}$, $V = \{X, Q, R, T, W\}$, and the production set P is:

$$\begin{aligned} X &\to aQ \mid bR \mid cW \\ Q &\to aR \mid bT \mid cX \\ R &\to aT \mid bW \mid cQ \mid \lambda \\ T &\to aW \mid bX \mid cR \\ W &\to aX \mid bQ \mid cT \mid \lambda \end{aligned}$$

Construct a state-transition graph of a finite automaton that accepts L. If such an automaton does not exist, prove it.

Answer: Apply the conversion algorithm:



Problem 4 [**10 points**] Let L be the set of exactly those strings over the alphabet $\Sigma = \{a, b, c\}$ where the total number of a's and c's is greater than 2 but not greater than 6.

Write a regular expression that defines L. If such a regular expression does not exist, prove it.

Answer:

$b^*(a \cup c)b^*(a \cup c)b^*(a \cup c)b^*(a \cup c \cup \lambda)b^*(a \cup c \cup \lambda)b^*(a \cup c \cup \lambda)b^*$

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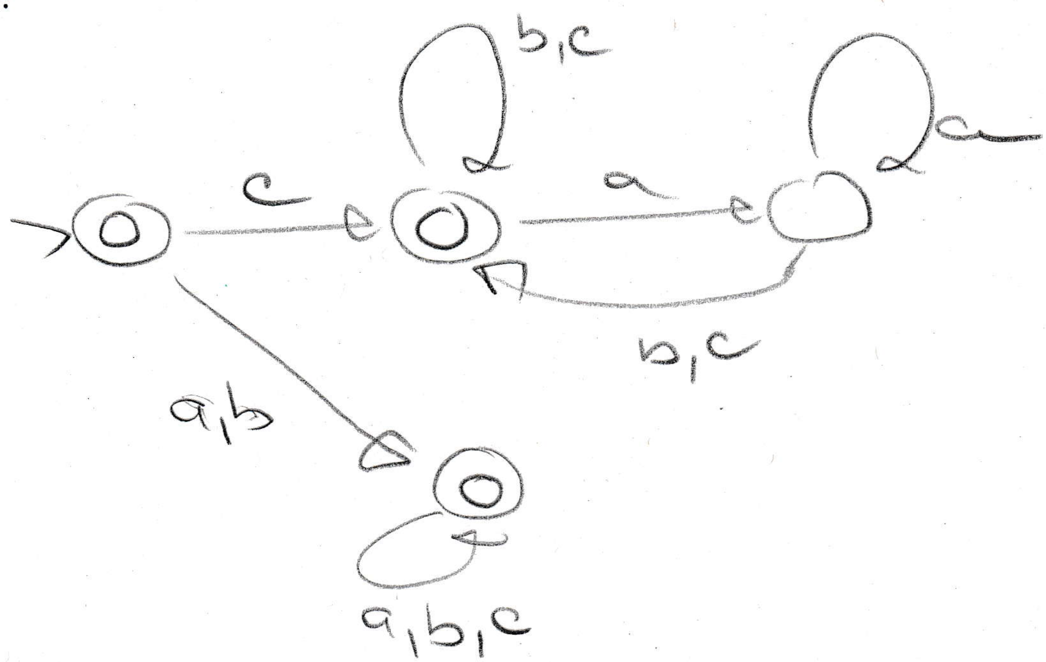
Problem 5 [**14 points**] Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ that begin with c and end with a.

Draw a state-transition graph of a finite automaton that accepts $\overline{L}$ (the complement of L). If such an automaton does not exist, prove it.

Answer:

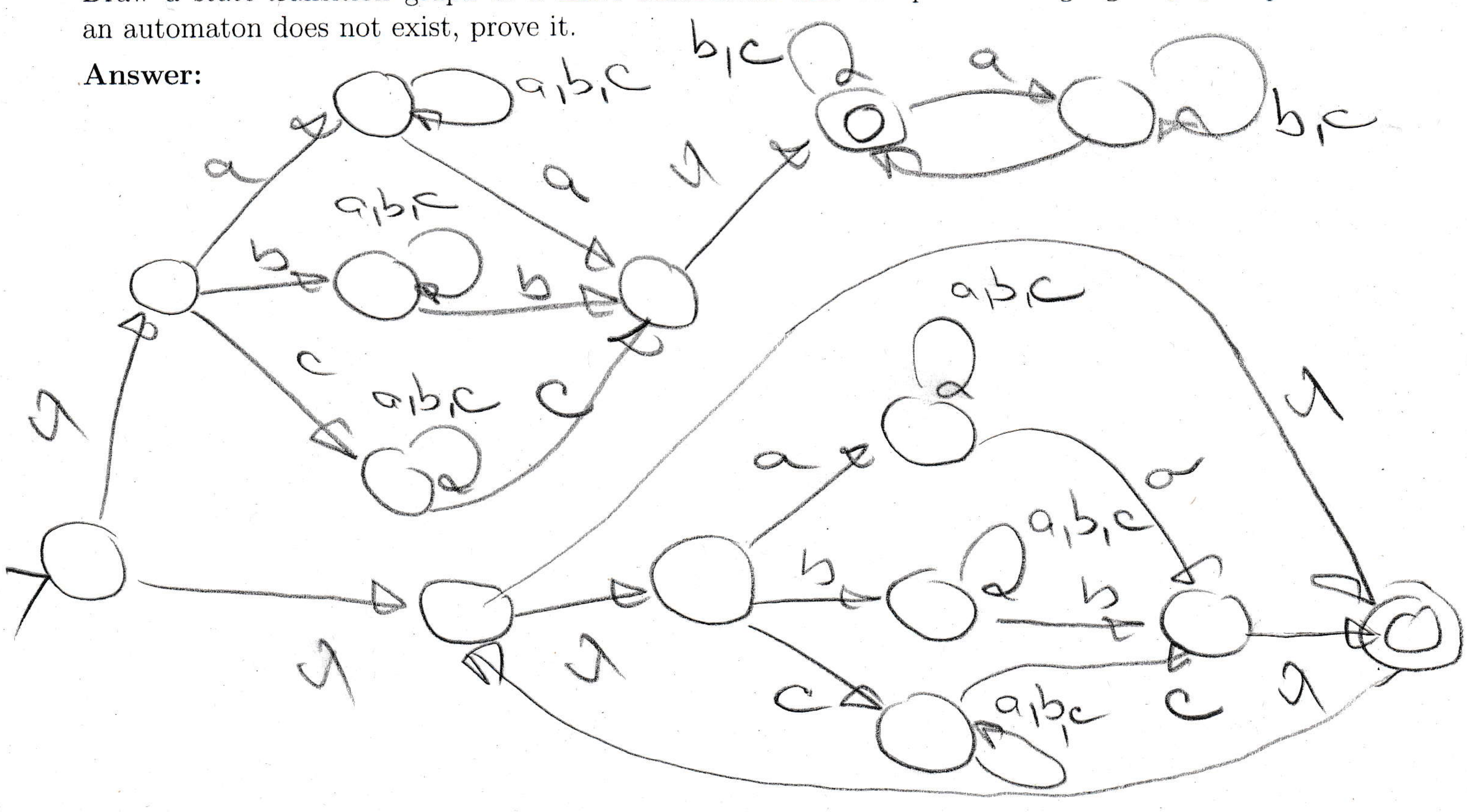


Problem 6 [**14 points**] Let L_1 be the set of exactly those strings over the alphabet $\{a, b\}$ whose length is greater than 1 and first symbol is equal to the last symbol.

Let L_2 be the set of strings over the alphabet $\{a, b\}$ that have an even number of a's.

Draw a state-transition graph of a finite automaton that accepts the language $L_1L_2 \cup L_1^*$. If such an automaton does not exist, prove it.

Answer:



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Problem 7 [**14 points**] Let L_1 be the set of non-empty palindromes of even length over the alphabet $\{a, b, c\}$.

Let L_2 be the set of palindromes of odd length over the alphabet $\{a, b, c\}$.

Write a complete formal definition of a context-free grammar that generates the language $(L_1 L_2)^* \cup (L_2 L_1)^*$. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$, $\Sigma = \{a, b, c\}$, $V = \{S, A, B, S_1, S_2\}$

P: $S \to A \mid B$

$A \to \lambda \mid AA \mid S_1 S_2$

$B \to \lambda \mid BB \mid S_2 S_1$

$S_1 \to aS_1a \mid bS_1b \mid cS_1c \mid aa \mid bb \mid cc$

$S_2 \to aS_2a \mid bS_2b \mid cS_2c \mid a \mid b \mid c$

Problem 8 m [**14 points**] Let L be the set of all strings of the form $a^k b^p a^\ell c^m b^j$ such that $k = j = 0, \ell > \cancel{\ell} + p$, where $k, m, j, \ell, p \geq 0$.

Write a complete formal definition of a context-free grammar that generates L. If such a grammar does not exist, prove it.

Answer: The template is: $b^p a^p a^{q+1} a^m c^m$

where $p, q, m \geq 0$, whence the grammar:

$G = (V, \Sigma, P, S)$, $\Sigma = \{a, b, c\}$, $V = \{S, A, B, D\}$

P: $S \to ABD$

$A \to bAa \mid \lambda$

$B \to aB \mid a$

$D \to aDc \mid \lambda$

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Problem 9 [**14 points**] Let L be the set of all strings of the form $a^k b^p a^\ell c^m b^j$ such that $k > m, p < \ell, j = 0$, where $k, m, j, \ell, p \geq 0$.

Write a complete formal definition of a context-free grammar that generates L. If such a grammar does not exist, prove it.

Answer: Template is: $a^{n+1} a^m b^p a^p a^{q+1} c^m$

where $m, n, p, q \geq 0$

whence the grammar: $G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$, $V = \{S, A, B, D\}$

P: $S \to AB$

$A \to aA \mid a$

$B \to aBc \mid DA$

$D \to bDa \mid \Lambda$

Problem 10 [**14 points**] Let L be the set of all strings of the form $a^k b^p a^\ell c^m b^j$ such that $k = \ell, p = j = 0$, where $k, m, j, \ell, p \geq 0$.

Write a regular expression that defines L. If such a regular expression does not exist, prove it.

Answer:

Template is: $a^{2k} c^m$, $k, m \geq 0$

whence the regular expression:

$(aa)^* c^*$

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Problem 11 [**16 points**] Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. the three letters occur in the string either in the alphabetic order or in the reversed alphabetic order;
2. each of the three letters occurs at least once;
3. if the letters occur in the alphabetic order, then there are more b's than c's;
4. if the letters occur in the reversed alphabetic order, then there are more a's than c's;

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, L, R, A, B, D, E\}$

$\Sigma = \{a, b, c\}$

P: $S \to L \mid R$

$L \to ABD$

$A \to aA \mid a$

$B \to bB \mid b$

$D \to bDc \mid bc$

$D \to EA$

$E \to cEa \mid cBa$